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# The Hamiltonian BRST quantization of massive abelian $p$-form gauge fields 

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#### Abstract

In this paper we quantize the massive abelian $p$-form gauge fields in $D$ dimensions in the context of the Hamiltonian BRST formalism. Extending the original phase-space in order to reveal the reducibility of a certain first-class system, we obtain that quantizing the original theory is the same as quantizing a ( $p-1$ )-order reducible system using BRST. This system describes abelian $p$-form gauge fields interacting through a current-current term with abelian ( $p-1$ )-form gauge fields. For $p=1$ and $D=4$ we recover the Stueckelberg coupling.


## 1. Introduction

The most powerful quantization method for gauge theories is well known to be the BRST formalism. For systems with both first- and second-class constraints, many BRST quantization alternatives have been implemented [1-6]. Some models studied intensively recently, especially from the Hamiltonian BRST point of view, are abelian $p$-form gauge fields free or coupled to vector fields [7-13]. The physical importance of these models consists on the one hand of the profound connection with string theory and, on the other, with cosmic strings, vortices and black holes [8, 12]. For theories possessing only secondclass constraints, this formalism cannot be directly applied because they are not gauge invariant. The BRST quantization of such theories can be achieved by turning the original second-class system into a first-class one (i) in the initial phase-space [14] or (ii) in a larger phase-space $[15,16]$, and then quantizing the resulting first-class systems in the framework of the BRST approach. Many authors [17-26] have applied the methods from [15, 16] to various models. Nevertheless, the method from [15, 16] has not yet been extended to the case of second-class systems preserving somehow the trace of reducibility of a certain first-class system and peculiarly to the case of massive abelian $p$-form gauge fields in $D$ dimensions. This is the purpose of this paper. In fact, in the Hamiltonian BRST formalism we shall quantize the massive abelian $p$-form gauge fields transforming the original system to a first-class one in the light of the procedures (i) and (ii) to be further discussed. Procedure (ii) will actually imply the BRST quantization of the $p$-form gauge fields interacting with ( $p-1$ ) ones through a current-current term. More on free abelian $p$-form gauge fields may be found in [7]. Related to the Hamiltonian BRST quantization we follow the same line as in [5].

Our starting point is the Lagrangian action
$S_{0_{p}}^{L}\left[A^{\mu_{1} \ldots \mu_{p}}\right]=\int \mathrm{d}^{D} x \frac{1}{2}\left(-\frac{1}{(p+1)!} F^{\mu_{1} \ldots \mu_{p+1}} F_{\mu_{1} \ldots \mu_{p+1}}-\frac{M^{2}}{p!} A^{\mu_{1} \ldots \mu_{p}} A_{\mu_{1} \ldots \mu_{p}}\right)$
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where $F_{\mu_{1} \ldots \mu_{p+1}}=\partial_{\left[\mu_{1}\right.} A_{\left.\mu_{2} \ldots \mu_{p+1}\right]}$ and $A_{\mu_{1} \ldots \mu_{p}}$ are antisymmetric in the Lorentz indices. Action (1) describes a purely second-class system with the canonical Hamiltonian

$$
\begin{align*}
H=\int \mathrm{d}^{D-1} x & \left((-)^{p+1} \frac{p!}{2} \pi_{i_{1} \ldots i_{p}} \pi_{i_{1} \ldots i_{p}}-p A_{0 i_{2} \ldots i_{p}} \partial_{i_{1}} \pi^{i_{1} i_{2} \ldots i_{p}}\right. \\
& \left.+\frac{M^{2}}{2 p!} A^{\mu_{1} \ldots \mu_{p}} A_{\mu_{1} \ldots \mu_{p}}+\frac{1}{2(p+1)!} F_{i_{1} \ldots i_{p+1}} F^{i_{1} \ldots i_{p+1}}\right) \tag{2}
\end{align*}
$$

and the primary, respectively, secondary second-class constraints

$$
\begin{align*}
G^{i_{2} \ldots i_{p}} & \equiv \pi^{0 i_{2} \ldots i_{p}}=0  \tag{3}\\
C^{i_{2} \ldots i_{p}} & \equiv p \partial_{i_{1}} \pi^{i_{1} i_{2} \ldots i_{p}}-\frac{M^{2}}{(p-1)!} A^{0 i_{2} \ldots i_{p}}=0 \tag{4}
\end{align*}
$$

where $\pi^{\mu_{1} \ldots \mu_{p}}$ are the canonical momenta conjugated to the $A_{\mu_{1} \ldots \mu_{p}}$ 's. We denote for later convenience $C_{(0)}^{i_{2} \ldots i_{p}} \equiv p \partial_{i_{1}} \pi^{i_{1} i_{2} \ldots i_{p}}$, and $C_{(1)}^{i_{2} \ldots i_{p}} \equiv-\frac{M^{2}}{(p-1)!} A^{0 i_{2} \ldots i_{p}}$. This completes the canonical analysis of our model.

## 2. The BRST quantization without extra fields

Because the second-class constraints (3) and (4) may be split into two equally numbered subsets such that the $G^{i_{2} \ldots i_{p}}$,s are first class among themselves, we can consider that the original system comes from a first-class one possessing only the constraints (3) in the canonical gauge (4). In this way, we obtain a first-class system in the original phase-space described by the Hamiltonian [14]

$$
\begin{equation*}
\bar{H}=H-\frac{(p-1)!}{2 M^{2}} \int \mathrm{~d}^{D-1} x C^{i_{2} \ldots i_{p}} C_{i_{2} \ldots i_{p}} \tag{5}
\end{equation*}
$$

which is first class with respect to the $G^{i_{2} \ldots i_{p}}$, s, i.e. $\left[\bar{H}, G^{i_{2} \ldots i_{p}}\right]=0$ strongly. Next, we quantize the last first-class system in the Hamiltonian BRST formalism. The BRST charge reads

$$
\begin{equation*}
\Omega=\int \mathrm{d}^{D-1} x\left(\eta^{i_{2} \ldots i_{p}} G_{i_{2} \ldots i_{p}}+\sum_{a=1}^{2} B_{i_{2} \ldots i_{p}}^{a} P_{\bar{\eta}^{a}}^{i_{2} \ldots i_{p}}\right) \tag{6}
\end{equation*}
$$

with $\eta^{i_{2} \ldots i_{p}}$,s the ghost fields and the remaining variables being auxiliary [27]. The BRSTinvariant extension of $\bar{H}$ is clearly BRST-invariant itself because $[\bar{H}, \Omega]=0$. We choose a gauge-fixing fermion implementing at the Hamiltonian BRST level the canonical gauge conditions (4). It has the form

$$
\begin{align*}
K=\int \mathrm{d}^{D-1} x & \left(\mathcal{P}_{i_{2} \ldots i_{p}}\left(\dot{A}^{0 i_{2} \ldots i_{p}}+C^{i_{2} \ldots i_{p}}\right)+P_{B^{2}}^{i_{2} \ldots i_{p}}\left(\mathcal{P}_{i_{2} \ldots i_{p}}-\bar{\eta}_{i_{2} \ldots i_{p}}^{1}+\dot{\bar{\eta}}_{i_{2} \ldots i_{p}}^{2}\right)\right. \\
& \left.+P_{B^{1}}^{i_{2} \ldots i_{p}}\left(\bar{\eta}_{i_{2} \ldots i_{p}}^{2}+\dot{\bar{\eta}}_{i_{2} \ldots i_{p}}^{1}\right)\right) \tag{7}
\end{align*}
$$

with $\mathcal{P}_{i_{2} \ldots i_{p}}$ the canonical momenta of the ghost fields, the other variables being associated to the auxiliary variables from (6). Computing the gauge-fixed action of the first-class system and integrating in the corresponding path integral, $Z_{K}$, over all the variables excepting the $A_{i_{1} \ldots i_{p}}$ 's and $\pi^{i_{1} \ldots i_{p}}$ 's, we get

$$
\begin{equation*}
Z_{K}=\int \mathcal{D} \pi^{i_{1} \ldots i_{p}} \mathcal{D} A_{i_{1} \ldots i_{p}} \operatorname{expi} \int \mathrm{~d}^{D} x\left(\dot{A}^{i_{1} \ldots i_{p}} \pi_{i_{1} \ldots i_{p}}-\bar{h}\right) \tag{8}
\end{equation*}
$$

where

$$
\begin{align*}
& \bar{h}=(-)^{p+1} \frac{p!}{2} \pi_{i_{1} \ldots i_{p}} \pi_{i_{1} \ldots i_{p}}+\frac{1}{2(p+1)!} F_{i_{1} \ldots i_{p+1}} F^{i_{1} \ldots i_{p+1}} \\
&+\frac{M^{2}}{2 p!} A_{i_{1} \ldots i_{p}} A^{i_{1} \ldots i_{p}}-\frac{p p!}{2 M^{2}}\left(\partial_{i_{1}} \pi^{i_{1} \ldots i_{p}}\right)^{2} \tag{9}
\end{align*}
$$

Equation (8) gives precisely the Hamiltonian path integral over independent variables for the first-class system. It coincides with the Hamiltonian path integral over independent variables of the original system. The validity of the prior statement may be checked by means of the canonical methods [28], confirming that our method is indeed correct. We can also derive the Lagrangian path integral for our model making in the gauge-fixed action the replacement $C_{i_{2} \ldots i_{p}} \rightarrow C_{i_{2} \ldots i_{p}}+\frac{M^{2}}{2(p-1)!} B_{i_{2} \ldots i_{p}}^{1}$, which does not affect its BRST invariances as $s B_{i_{2} \ldots i_{p}}^{1}=0$. Then, we deduce

$$
\begin{equation*}
Z_{K}^{L}=\int \mathcal{D} A_{i_{1} \ldots i_{p}} \operatorname{det}(M) \exp i S_{0_{p}}^{L} \tag{10}
\end{equation*}
$$

Relations (8) and (10) are the main results of this section.

## 3. The BRST quantization with extra fields

In this section we quantize our model converting the original second-class constraints into some first-class ones by adding some extra fields and their conjugated momenta, and subsequently applying the Hamiltonian BRST approach. Here, we shall not use the BFT method $[15,16]$, but a different way which we extend to include the 'reducible' case. The above notion of 'reducibility' will be made clear below. For the purpose of this section, it is convenient to change the Hamiltonian (5) slightly, such that the new Hamiltonian remains in the same gauge-invariant function equivalence class as $\bar{H}$. Thus, we take as the new Hamiltonian in the original phase-space, the function

$$
\begin{equation*}
H^{\prime}=H-\frac{(p-1)!}{M^{2}} \int \mathrm{~d}^{D-1} x\left(C_{(0)}^{i_{2} \ldots i_{p}} C_{(1)^{i_{2} \ldots i_{p}}}+\frac{1}{2} C_{(1)}^{i_{2} \ldots i_{p}} C_{\left.(1)^{i_{2} \ldots i_{p}}\right)}\right) \tag{11}
\end{equation*}
$$

Our second conversion method is realized in two steps. At the first step, we associate as in section 2, to the original system a first-class one in the original phase-space with the firstclass Hamiltonian (11) and the first-class constraints (3). At the second step, we associate to the last system a one-parameter family of first-class systems in a larger phase-space constructed as follows.
(i) For every pair $\left(G_{i_{2} \ldots i_{p}}, C_{(0)}^{i_{2} \ldots i_{p}}\right)$ we introduce the bosonic canonical pair $\left(V^{i_{1} \ldots i_{p-1}}, \Pi_{i_{1} \ldots i_{p-1}}\right)$ with the new variables antisymmetric in their indices, such that the new secondary constraints are

$$
\begin{equation*}
\gamma_{i_{2} \ldots i_{p}} \equiv \lambda \Pi_{i_{2} \ldots i_{p}}-C_{(0) i_{2} \ldots i_{p}}=0 \tag{12}
\end{equation*}
$$

where $\lambda$ is the non-vanishing parameter. We see simply that

$$
\begin{equation*}
\partial^{i_{2}} \gamma_{i_{2} \ldots i_{p}}=\lambda \partial^{i_{2}} \Pi_{i_{2} \ldots i_{p}}=0 \tag{13}
\end{equation*}
$$

(ii) For every relation (13) we add the new bosonic canonical pair of fields $\left(q^{0 i_{2} \ldots i_{p-1}}, p_{0 i_{2} \ldots i_{p-1}}\right)$, also antisymmetric in their indices together with the supplementary constraints

$$
\begin{equation*}
\bar{G}_{i_{2} \ldots i_{p-1}} \equiv p_{0 i_{2} \ldots i_{p-1}}=0 \tag{14}
\end{equation*}
$$

such that the consistency of (14) implies as new constraints (up to a factor) relations (13). Following this procedure, we built starting with the original model, a first-class family in a larger phase-space possessing only the first-class constraints (3), (12), (14) and

$$
\begin{equation*}
\bar{\gamma}_{i_{2} \ldots i_{p-1}} \equiv-(p-1) \partial^{i_{1}} \Pi_{i_{1} \ldots i_{p-1}}=0 \tag{15}
\end{equation*}
$$

A beautiful feature of this constraints is that they are reducible this time. Indeed, the following reducibility relations hold:

$$
\begin{align*}
& \lambda \bar{\gamma}_{i_{2} \ldots i_{p-1}}+(p-1) \partial^{i_{1}} \gamma_{i_{1} \ldots i_{p-1}}=0  \tag{16}\\
& \partial^{i_{2}} \bar{\gamma}_{i_{2} \ldots i_{p-1}}=0 . \tag{17}
\end{align*}
$$

The reducibility functions appearing in (16), (17) are further reducible, namely

$$
\begin{equation*}
Z_{a_{k-1}}^{a_{k-2}} Z_{a_{k}}^{a_{k-1}}=0 \tag{18}
\end{equation*}
$$

with

$$
Z_{a_{k}}^{a_{k-1}}=\left(\begin{array}{cc}
Z_{j_{1} \ldots j_{p-k-1}}^{i_{1} \ldots i_{p-k}} & 0  \tag{19}\\
\bar{Z}_{j_{1} \ldots j_{p-k-1}}^{i_{1} \ldots i_{p-k-1}} & Z_{j_{1} \ldots j_{p-k-2}}^{i_{1} \ldots i_{p-k-1}}
\end{array}\right)
$$

and

$$
\begin{align*}
Z_{j_{1} \ldots j_{p-k-1}}^{i_{1} \ldots i_{p-k}} & =\frac{1}{(p-k)!} \delta^{\left[i_{1}\right.}{ }_{j_{1}} \ldots \delta_{j_{p-k-1}}^{i_{p-k-1}} \partial^{\left.i_{p-k}\right]}  \tag{20}\\
\bar{Z}_{j_{1} \ldots . \ldots-k-1}^{i_{1} \ldots i_{p-k-1}} & =\frac{\lambda}{(p-k-1)!} \delta^{\left[i_{1}\right.} \ldots \delta_{j_{1}}^{\left.i_{p-k-1}\right]} \tag{21}
\end{align*}
$$

where $k$ takes values from 1 to $(p-1)$. Now, it is clear that the first-class family is ( $p-1$ )-order reducible.

With step (ii) at hand, we build the first-class Hamiltonian of the first-class family of the form

$$
\begin{align*}
H^{*}=H^{\prime}+\int & \mathrm{d}^{D-1} x\left(-\frac{\lambda^{2}(p-1)!}{2 M^{2}} \Pi_{i_{1} \ldots i_{p-1}} \Pi^{i_{1} \ldots i_{p-1}}\right. \\
& \left.+A^{0 i_{2} \ldots i_{p}} \gamma_{i_{2} \ldots i_{p}}+q^{0 i_{2} \ldots i_{p-1}} \bar{\gamma}_{i_{2} \ldots i_{p-1}}+g\right) \tag{22}
\end{align*}
$$

where $g$ is a function satisfying

$$
\begin{align*}
& {\left[G_{i_{2} \ldots i_{p}}, g\right]=0 \quad\left[\bar{G}_{i_{2} \ldots i_{p-1}}, g\right]=0}  \tag{23}\\
& {\left[\gamma_{i_{2} \ldots i_{p}}, H^{*}\right]=0}  \tag{24}\\
& {\left[\bar{\gamma}_{i_{2} \ldots i_{p-1}}, g\right]=0} \tag{25}
\end{align*}
$$

We solve the system (23)-(25) representing $g$ as a series of powers in $V^{i_{1} \ldots i_{p-1}}$ with coefficients depending only on the $A^{i_{1} \ldots i_{p}}$,s

$$
\begin{equation*}
g=\stackrel{(1)}{g}_{i_{2} \ldots i_{p}} V^{i_{2} \ldots i_{p}}+\stackrel{(2)}{g}_{i_{2} \ldots i_{p}}^{i_{2} \ldots i_{p}} V_{i_{2} \ldots i_{p}} V^{i_{2} \ldots i_{p}}+\cdots \tag{26}
\end{equation*}
$$

With this choice, equations (23) are automatically verified. Introducing (26) in (24), we get

$$
\begin{align*}
& \stackrel{(1)}{g}_{i_{2} \ldots i_{p}}=\frac{M^{2}}{\lambda(p-1)!} \partial^{i_{1}} A_{i_{1} i_{2} \ldots i_{p}}  \tag{27}\\
& \stackrel{(2)}{g}_{i_{2} \ldots i_{p} \ldots i_{p}}^{i_{2}}=-\frac{p}{2 \lambda}\left[\partial_{i_{1}} \pi^{i_{1} \ldots i_{p}}, \stackrel{(1)}{g}_{j_{2} \ldots j_{p}}\right] \tag{28}
\end{align*}
$$

all the other coefficients being equal to zero. Using equations (27), (28) it follows that (25) is also satisfied. Thus, the function $g$ reads

$$
\begin{equation*}
g=-\frac{M^{2}}{\lambda \cdot p!}\left(A_{i_{1} \ldots i_{p}}-\frac{1}{2 \lambda} \tilde{F}_{i_{1} \ldots i_{p}}\right) \tilde{F}^{i_{1} \ldots i_{p}} \tag{29}
\end{equation*}
$$

where $\tilde{F}_{i_{1} \ldots i_{p}}=\partial_{\left[i_{1}\right.} V_{\left.i_{2} \ldots i_{p}\right]}$.
The next step of our analysis consists in quantizing the first-class family in light of the Hamiltonian BRST. In this case we are dealing with a ( $p-1$ )-order reducible theory revealing a more complicated structure than that of abelian free $p$-form gauge fields. This is because here there appear two sets of secondary constraints and also two sets of reducibility relations for them (see equations (16) and (17)). This fact further implies two types of ghosts of ghosts, ghosts of ghosts of ghosts, etc. The BRST charge in this case reads

$$
\begin{align*}
\Omega^{\prime}=\int \mathrm{d}^{D-1} x & \left(G_{i_{1} \ldots i_{p-1}} \eta_{1}^{i_{1} \ldots i_{p-1}}+\bar{G}_{i_{1} \ldots i_{p-2}} \mathcal{C}_{1}^{i_{1} \ldots i_{p-2}}+\gamma_{i_{1} \ldots i_{p-1}} \eta_{2}^{i_{1} \ldots i_{p-1}}+\bar{\gamma}_{i_{1} \ldots i_{p-2}} \mathcal{C}_{2}^{i_{1} \ldots i_{p-2}}\right. \\
& +\sum_{a=1}^{2} B_{i_{1} \ldots i_{p-1}}^{a} P_{\bar{\eta}^{a}}^{i_{1} \ldots i_{p-1}}+\sum_{k=1}^{p-1} \mathcal{P}_{i_{1} \ldots i_{p-k}}^{2} \partial^{i_{p-k}} \eta_{2}^{i_{1} \ldots i_{p-k-1}} \\
& \left.+\sum_{k=1}^{p-1} \overline{\mathcal{P}}_{i_{1} \ldots i_{p-k-1}}^{2}\left(\partial^{i_{p-k-1}} \mathcal{C}_{2}^{i_{1} \ldots i_{p-k-2}}+\lambda \eta_{2}^{i_{1} \ldots i_{p-k-1}}\right)\right) \tag{30}
\end{align*}
$$

The BRST-invariant extension of the first-class Hamiltonian (22) is given by

$$
\begin{equation*}
H_{\mathrm{B}}=H^{*}+\int \mathrm{d}^{D-1} x\left(\overline{\mathcal{P}}_{i_{1} \ldots i_{p-2}}^{2} \mathcal{C}_{1}^{i_{1} \ldots i_{p-2}}+\mathcal{P}_{i_{1} \ldots i_{p-1}}^{2} \eta_{1}^{i_{1} \ldots i_{p-1}}\right) \tag{31}
\end{equation*}
$$

The gauge-fixing fermion implementing the canonical gauge conditions $V_{i_{1} \ldots i_{p-1}}=0$ and $q_{0 i_{2} \ldots i_{p-1}}=0$ takes the form

$$
\begin{align*}
K^{\prime}=\int \mathrm{d}^{D-1} x & \left(\mathcal{P}_{i_{1} \ldots i_{p-1}}^{1}\left(\dot{A}^{0 i_{1} \ldots i_{p-1}}+\frac{1}{\lambda} V^{i_{1} \ldots i_{p-1}}\right)+\overline{\mathcal{P}}_{i_{1} \ldots i_{p-2}}^{1}\left(\dot{q}^{0 i_{1} \ldots i_{p-2}}+\frac{1}{\lambda} q^{0 i_{1} \ldots i_{p-2}}\right)\right. \\
& +P_{B^{2}}^{i_{1} \ldots i_{p-1}}\left(\bar{\eta}_{i_{1} \ldots i_{p-1}}^{1}+\dot{\bar{\eta}}_{i_{1} \ldots i_{p-1}}^{2}\right)+P_{B^{1}}^{i_{1} \ldots i_{p-1}}\left(\mathcal{P}_{i_{1} \ldots i_{p-1}}^{1}-\bar{\eta}_{i_{1} \ldots i_{p-1}}^{2}+\dot{\bar{\eta}}_{i_{1} \ldots i_{p-1}}^{1}\right) \\
& \left.-\frac{1}{\lambda} \sum_{k=1}^{p-1} \mathcal{P}_{i_{1} \ldots i_{p-k-1}}^{2}\left(-\dot{\mathcal{C}}_{2}^{i_{1} \ldots i_{p-k-1}}+\mathcal{C}_{2}^{i_{1} \ldots i_{p-k-1}}\right)\right) \tag{32}
\end{align*}
$$

In equations (30)-(32) we denoted by $\left(\mathcal{C}_{a}^{i_{1} \ldots i_{p-2}}, \eta_{a}^{i_{1} \ldots i_{p-1}}\right)_{a=1,2}$ the ghost fields corresponding to the first-class constraints, while $\left(\mathcal{P}_{i_{1} \ldots i_{p-1}}^{a}, \mathcal{P}_{i_{1} \ldots i_{p-1}}^{a}\right)_{a=1,2}$ are, respectively, their conjugated momenta. The ghost numbers and Grassmann parities of the above fields are all equal to one, respectively one. The fields $\left(\mathcal{C}_{a}^{i_{1} \ldots i_{p-k-2}}, \eta_{a}^{i_{1} \ldots i_{p-k-1}}\right)_{a=1,2 ; k=1, \ldots, p-1}$ are ghost fields with ghost numbers $(k+1)$ and Grassmann parities $(k+1) \bmod 2$, while $\left(\overline{\mathcal{P}}_{i_{1} \ldots i_{p-k-2}}^{a}, \mathcal{P}_{i_{1} \ldots i_{p-k-1}}^{a}\right)_{a=1,2 ; k=1, \ldots, p-1}$ stand for their momenta and have the ghost numbers $-(k+1)$ and Grassmann parities $(k+1) \bmod 2$. The remaining fields in (30)-(32) form the non-minimal sector (32). The notations employed in formulae (30)-(32) must be understood as $f^{j_{1} \ldots j_{p-k-2}}=0$, for $p-k-2<0$ and $f^{j_{1} \ldots j_{p-k-2}}=f$, for $p-k-2=0$. Integrating in the path integral, $Z_{K^{\prime}}^{\prime}$, constructed with the gauge-fixed action corresponding to the first-class family over all the fields excepting the $A^{i_{1} \ldots i_{p}}$ 's and $\pi_{i_{1} \ldots i_{p}}$ 's, we deduce

$$
\begin{equation*}
Z_{K^{\prime}}^{\prime}=Z_{K} \tag{33}
\end{equation*}
$$

where $Z_{K}$ is given in (8). The Lagrangian path integral in this method is inferred by making in the gauge-fixed action of the first-class family the replacements $\Pi_{i_{2} \ldots i_{p}} \rightarrow$
$\Pi_{i_{2} \ldots i_{p}}+\frac{1}{\lambda} C_{(0) i_{2} \ldots i_{p}}$ (which do not affect its BRST invariances) and subsequently integrating in the resulting path integral over all the fields excepting the $A^{\mu_{1} \ldots \mu_{p}}$ 's. Thus, we find the same Lagrangian path integral as in (10).

Now, it appears clearly the meaning of the original system preserving the trace of reducibility of a certain first-class system. This first-class system is actually described by the first-class Hamiltonian (22) together with the first-class constraints (3), (12), (14), (15). Indeed, if one takes all the extra fields equal to zero in (22), one reobtains precisely the original system. At the same time, the gauge-fixing fermion (32) implying the vanishing of all extra fields leads to the same path integral of the first-class family as the original one. In this sense, the original second-class system maintains the reducibility relic of the first-class family, which is a truly $(p-1)$-order reducible theory.

## 4. The Lagrangian action of the first-class family

In this section we will explain the origin of the first-class family constructed in section 3. Eliminating from the total action [5] corresponding to the Hamiltonian (22) and to the constraints (3), (14) all the momenta and Lagrange multipliers on their equations of motion [27], we arrive at the following Lagrangian action of the first-class family

$$
\begin{array}{rl}
S_{0_{p, p-1}}^{L}=\int \mathrm{d}^{D} & x\left(-\frac{1}{2(p+1)!} F^{\mu_{1} \ldots \mu_{p+1}} F_{\mu_{1} \ldots \mu_{p+1}}\right. \\
& \left.-\frac{M^{2}}{2 p!}\left(A^{\mu_{1} \ldots \mu_{p}}-\frac{1}{\lambda} \tilde{F}^{\mu_{1} \ldots \mu_{p}}\right)\left(A_{\mu_{1} \ldots \mu_{p}}-\frac{1}{\lambda} \tilde{F}_{\mu_{1} \ldots \mu_{p}}\right)\right) \tag{34}
\end{array}
$$

where $\tilde{F}_{\mu_{1} \ldots \mu_{p}}=\partial_{\left[\mu_{1}\right.} A_{\left.\mu_{2} \ldots \mu_{p}\right]}$ and $A_{\mu_{1} \ldots \mu_{p-1}}=\left(q_{0 i_{2} \ldots i_{p-1}}, V_{i_{1} \ldots i_{p-1}}\right)$. Action (34) is invariant under the next gauge transformations

$$
\begin{align*}
& \delta_{\epsilon} A_{\mu_{1} \ldots \mu_{p}}=\partial_{\left[\mu_{1}\right.} \epsilon_{\left.\mu_{2} \ldots \mu_{p}\right]}  \tag{35}\\
& \delta_{\epsilon} A_{\mu_{1} \ldots \mu_{p-1}}=\partial_{\left[\mu_{1}\right.} \epsilon_{\left.\mu_{2} \ldots \mu_{p-1}\right]}+\lambda \epsilon_{\mu_{1} \ldots \mu_{p-1}} \tag{36}
\end{align*}
$$

with $\epsilon$ 's antisymmetric arbitrary functions. The system described by action (34) comes from the gauging of the rigid (Noether) symmetries $\delta_{\epsilon} A_{\mu_{1} \ldots \mu_{p-1}}=\lambda \epsilon_{\mu_{1} \ldots \mu_{p-1}}$ (here $\epsilon_{\mu_{1} \ldots \mu_{p-1}}$ are all constant) of the action

$$
\begin{equation*}
S_{0_{p-1}}^{L}=\int \mathrm{d}^{D} x\left(-\frac{M^{2}}{2 \lambda^{2} p!} \tilde{F}^{\mu_{1} \ldots \mu_{p}} \tilde{F}_{\mu_{1} \ldots \mu_{p}}\right) \tag{37}
\end{equation*}
$$

If we now make the $\epsilon_{\mu_{1} \ldots \mu_{p-1}}$ 's functions of $x$, then action (37) is no longer gauge invariant. In order to render it invariant it is necessary to couple the $\tilde{F}^{\mu_{1} \ldots \mu_{p}}$, s with a tensor field transforming as in (35) such that the objects $A^{\mu_{1} \ldots \mu_{p}}-\frac{1}{\lambda} \tilde{F}^{\mu_{1} \ldots \mu_{p}}$ become gauge-invariant. Constructing with these objects a Lorentz scalar as in (34), we observe that we can add to it an other gauge-invariant scalar, namely the first term in (34). Following our procedure, the first-class family associated to the massive $p$-form gauge fields describes $p$-form gauge fields in interaction with $(p-1)$-form gauge fields by means of a current-current type term, where the gauge-invariant conserved current is

$$
\begin{equation*}
j_{\mu_{2} \ldots \mu_{p}}^{\mu_{1}}=\frac{M}{p!}\left(A_{\mu_{2} \ldots \mu_{p}}^{\mu_{1}}-\frac{1}{\lambda} \tilde{F}_{\mu_{2} \ldots \mu_{p}}^{\mu_{1}}\right) . \tag{38}
\end{equation*}
$$

The current (38) corresponds via Noether's theorem to the above rigid symmetries for action (34). At the Lagrangian level, we are able to fix the value of the parameter comparing the path integral (10) with the one derived from (34) using the antifield formalism. We find $\lambda=M$.

For $p=1$ and $D=4$, action (34) reduces to

$$
\begin{equation*}
S_{0_{1,0}}^{L}=\int \mathrm{d}^{4} x\left(-\frac{1}{4} F^{\mu \nu} F_{\mu \nu}-\frac{1}{2}\left(M A^{\mu}-\partial^{\mu} \varphi\right)\left(M A_{\mu}-\partial_{\mu} \varphi\right)\right) \tag{39}
\end{equation*}
$$

describing an irreducible theory which is identical to the one obtained in [29], and also in [30] using a conversion method for irreducible systems.

In conclusion, we can quantize consistently the massive abelian $p$-form gauge fields in the framework of the Hamiltonian BRST formalism in the original phase-space, as well as in a larger one. The last quantization implies in fact the quantization of a ( $p-1$ )-order reducible theory expressing $p$-form gauge fields interacting with $(p-1)$-form gauge fields through a current-current term.

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